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System dynamics model of high-power LED luminaire

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ABSTRACT

Optical properties of LEDs are sensitive to junction temperature. From the principle of solid-state lighting, the luminance of LED is induced from two physical mechanisms: energy effect and optoelectronic effect. Both effects are related to junction temperature. The understanding of system dynamic behavior in junction temperature is quite important for lighting control design.

The system dynamics model of a high-power LED fixture for energy effect was derived and identified in the present study using step response method. Both theoretical and experimental analyses have shown that the thermal system dynamics model of the LED fixture is 4th-order with three zeros and can be reduced to a first-order biproper system. The instantaneous jump of junction temperature dominates the thermal behavior of LED at the beginning of the step input. The optoelectronic effect was induced mainly from the current input and the junction temperature. Combining the two physical effects, an electric-heat-optical system dynamics model of LED luminaire was finally proposed which is the basic system dynamics model for LED luminance control.

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1. Introduction

High-power LED is a promising technology for future lighting application since it can save energy and has a long lifetime. The light output intensity and wavelength, light decay, and color change are sensitive to the junction temperature of LED. The LED junction temperature depends on the power input, the environmental conditions (ambient temperature, humidity, and wind), the heat sink and the lighting fixture design. For white LEDs, junction temperature variation will cause light intensity change and even permanent deterioration [1]. Similarly, for polychromatic LEDs, the lighting characteristics, e.g. color temperature shift and luminance decay, are also related with its junction temperature [2].

Many investigators have studied the lighting control of multi-color LEDs. Several researchers focused on lighting property control by luminance feedback [3–6]. Few researchers [7,8] proposed temperature feedback using steady-state thermal resistance.

The junction temperature variation is affected by LED lamp packaging, luminaire design, input power variation, and environmental conditions (wind speed, ambient temperature and humidity etc.). Therefore, the key for good quality of light output control is to obtain an accurate system dynamics model of the LED luminaire that can accurately predict the junction temperature variations.

Huang et al. [9] have studied system dynamics model for the heat transfer behavior of LED luminaire and used them in the current control of LED luminaire. In the derivation of the system dynamics model for current control [10] the major concern is the current variation induced by the variations of applied voltage of LED and ambient temperature.

From the principle of solid-state lighting, the luminance of LED is induced from two physical mechanisms: energy effect and optoelectronic effect. Both effects are related to junction temperature. The optoelectronic effect of a LED is induced mainly from the current input and the junction temperature. An electric-heat-optical system dynamics model of LED luminaire thus needs to be derived to describe the complete physical phenomenon of an LED lighting process, an electric-heat-optical effect.

The present paper presents in more details the theoretical derivation of the system dynamics model of a LED luminaire to study the affects of LED driving current and ambient temperature variation on junction temperature. The model parameters are identified experimentally using a special LED junction temperature measuring technique. An electric-heat-optical system dynamics model of LED luminaire is derived to describe the complete physical phenomenon of an LED lighting process, an electric-heat-optical effect.

This paper is arranged in the following sections. In Section 2, the system dynamics model of LED luminaire is theoretically derived as an MIMO linear system. Section 3 discusses the luminaire design using white LED lamps and methodology of junction temperature measurement. Section 4 presents the model parameters identification. Section 5 discusses the electric-heat-optical effect in junction temperature response from the derived system dynamics model.

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Nomenclature

A	section area of the diode (m^2)
A_{led}	A matrix of LED luminaire
a_0	relative bias voltage (V)
a_1	temperature sensitive parameter ($\text{V}/^\circ\text{C}$)
B_{led}	B matrix of LED luminaire
C	equivalent heat capacity ($\text{J}/\text{kg K}$)
C_{led}	C matrix of LED luminaire
D_e	electrons diffusion coefficient (m^2/s)
D_h	holes diffusion coefficient (m^2/s)
E_{go}	energy gap at zero Kelvin (J)
$G(s)$	system dynamics model ($^\circ\text{C}/\text{W}$)
$G_{\text{LED}}(s)$	system dynamics model of luminaire ($^\circ\text{C}/\text{W}$)
$H_1(s)$	biproper system dynamics model ($^\circ\text{C}/\text{A}$)
$H_4(s)$	4th-order system dynamics model ($^\circ\text{C}/\text{A}$)
$H_{\text{pj}}(s)$	equivalent system dynamics model for input current to junction temperature ($^\circ\text{C}/\text{A}$)
$H_{\text{th}}(s)$	thermal model ($^\circ\text{C}/\text{A}$)
I_f	forward current (A)
I_s	saturation current (A)
K	overall thermal conductivity ($\text{W}/\text{m K}$)
K_i	constant of intrinsic concentration equation
k	gain of system dynamics model ($^\circ\text{C}/\text{mA}$)
k_b	Boltzman constant (J/K)
L_e	electrons diffusion length (m)
L_h	holes diffusion length (m)
M	equivalent mass (kg)
N_A	acceptors concentration (m^{-3})
N_D	donors concentration (m^{-3})
n_i	intrinsic concentration of carriers (m^{-3})

P_d	heat power (W)
p	pole of system dynamics model
$\dot{Q}$	heat transfer rate (W)
q	electric charge (C)
R	thermal resistance (K/W)
T	lump temperature at specific part (K or $^\circ\text{C}$)
V_f	forward voltage (V)
z	zero of system dynamics model

Subscripts

0	initial time
a	ambient air
b	heat sink of luminaire
c	capsule of package
j	junction of LED chip
s	heat slug of package
p	input power

Greek symbols

γ	instantaneous jump rate ($^\circ\text{C}/\text{A}$)
δ	proportional constant for junction temperature to luminance ($\text{lm}/^\circ\text{C}$)
ε	proportional constant for input current to luminance (lm/A)
τ	time constant (s)
Φ	luminance of LED luminaire (lm)
$\sim$	perturbation
–	average

2. System dynamics modeling of LED luminaire

A white LED lamp consists of three basic elements: chip, heat slug, and capsule with phosphor glue [11]. The LED lamps, the metal core PCB (MCPCB), and the heat sink then make up the light engine of a luminaire. According to the thermal process, the luminaire can be divided into four major parts: (1) Junction – including whole chip structure such as die, bump, and submount; (2) Capsule – including optical lens, silicone glue, and phosphor; (3) Heat slug: including the heat conducting block and optical reflector, which is embedded below the LED junction for conducting heat to the heat sink; (4) Heat sink: which is attached to the heat slug, including MCPCB and heat sink.

Junction is the heat source in LED. When the electrical power is applied to the chip, there are two mechanisms in semiconductor, namely radiative and non-radiative recombination [2]. In the radiative combination, the photons are emitted as light. During non-radiative recombination, the electron energy is converted to vibrational energy of lattice atoms, i.e. phonons. Thus, the electron energy is converted to heat at the junction which is conducted to the adjacent parts. Capsule is in contact with junction, but the heat transfer to ambient is small and negligible due to small convective surface.

The heat slug has high conductivity with relatively small mass and provides major heat transfer from the junction. Heat sink is designed with high conductivity material whose large surface can dissipate heat to the ambient. The overall heat transfer of the LED luminaire can be described using the thermal-network model in Fig. 1. The temperatures of each part are defined as a single value (thermally lumped) and disregarding temperature gradient. The dynamic model for the heat transfer of thermally-lumped components can be derived by using the energy conservation principle as Eqs. (1)–(4) [12].

Junction:

$$M_j C_j \dot{T}_j = P_d - \dot{Q}_{jc} - \dot{Q}_{js} = P_d - K_{jc}(T_j - T_c) - K_{js}(T_j - T_s) \quad (1)$$

Capsule:

$$M_c C_c \dot{T}_c = \dot{Q}_{jc} + \dot{Q}_{sc} = K_{jc}(T_j - T_c) + K_{sc}(T_s - T_c) \quad (2)$$

Heat Slug:

$$M_s C_s \dot{T}_s = \dot{Q}_{jc} - \dot{Q}_{sc} - \dot{Q}_s = K_{jc}(T_j - T_c) - K_{sc}(T_s - T_c) - K_{sb}(T_s - T_b) \quad (3)$$

Heat Sink:

$$M_b C_b \dot{T}_b = \dot{Q}_{sb} - \dot{Q}_{ba} = K_{sb}(T_s - T_b) - K_{ba}(T_b - T_a) \quad (4)$$

where M 's is the equivalent mass, C 's is the equivalent heat capacity, $\dot{Q}$'s is the heat transfer rate and K 's is the equivalent thermal conductivity. K 's can represent overall heat conduction characteristics including the effects of contact surface area, roughness, thermal interface material, and contact pressure etc.

According to the above dynamics model, the information-flow diagram for the thermal process of a LED luminaire can be described by Fig. 2. The thermal process of a LED luminaire is a Multi-Input Multi-Output (MIMO) system with two inputs (input heat power P_d and ambient temperature T_a) and four outputs (T_j , T_c , T_s , T_b).

The dynamics model, Eqs. (1)–(4), is nonlinear and needs to be converted into a linear model around the steady operating point using linear-perturbation principle. The linear state-space model, Eqs. (5)–(7), is first obtained and then converted into a transfer-function model as shown in Eq. (8), using the relation $G(s) = C(sI - A)^{-1}B + D$. Therefore, the information-flow-diagram can be redraw as the system block diagram shown in Fig. 3. It can represent the input/output relationships of the LED system.

Fig. 1. Thermal network model of a LED luminaire.

Fig. 2. The information-flow-diagram of LED luminaire.

Fig. 3. MIMO system block diagram of a LED luminaire.

$$\dot{\tilde{\mathbf{x}}}_{\text{LED}} = \mathbf{A}_{\text{LED}}\tilde{\mathbf{x}}_{\text{LED}} + \mathbf{B}_{\text{LED}}\tilde{\mathbf{u}}_{\text{LED}} \quad (5)$$

$$\tilde{\mathbf{y}}_{\text{LED}} = \mathbf{C}_{\text{LED}}\tilde{\mathbf{x}}_{\text{LED}} \quad (6)$$

$$\tilde{\mathbf{x}}_{\text{LED}} = \begin{bmatrix} \tilde{T}_j \\ \tilde{T}_c \\ \tilde{T}_s \\ \tilde{T}_b \end{bmatrix}, \quad \tilde{\mathbf{u}}_{\text{LED}} = \begin{bmatrix} \tilde{P}_d \\ \tilde{T}_a \end{bmatrix}, \quad \tilde{\mathbf{y}}_{\text{LED}} = \begin{bmatrix} \tilde{T}_j \\ \tilde{T}_c \\ \tilde{T}_s \\ \tilde{T}_b \end{bmatrix} \quad (7)$$

$$\mathbf{G}_{\text{LED}}(s) = \mathbf{C}_{\text{LED}}(s\mathbf{I} - \mathbf{A}_{\text{LED}})^{-1}\mathbf{B}_{\text{LED}} = \begin{bmatrix} G_{pj}(s) & G_{aj}(s) \\ G_{pc}(s) & G_{ac}(s) \\ G_{ps}(s) & G_{as}(s) \\ G_{pb}(s) & G_{ab}(s) \end{bmatrix} \quad (8)$$

The element of $G_{pj}(s)$ in Eq. (8) is the most important model which describes the variation of junction temperature $\tilde{T}_j$ caused by input power $\tilde{P}_d$. $G_{pj}(s)$ can be derived from the state-space model and ex-

pressed in Eq. (9). It is seen that the LED luminaire is basically a 4th-order dynamic system with three zeros

$$G_{pj}(s) \equiv \frac{\tilde{T}_j(s)}{\tilde{P}_d(s)} = \frac{1}{\det(s\mathbf{I} - \mathbf{A}_{\text{LED}})} \left\{ s^3 + \left(\frac{K_{jc} + K_{sc}}{M_c C_c} + \frac{K_{js} + K_{sb} + K_{sc}}{M_s C_s} + \frac{K_{sb}}{M_b C_b} \right) s^2 + \left[\frac{(K_{jc} + K_{sc})(K_{js} + K_{sb} + K_{sc}) - K_{sc}^2}{M_c C_c M_s C_s} + \frac{K_{sb}(K_{js} + K_{sb} + K_{sc}) - K_{sc}^2}{M_s C_s M_b C_b} \right] s + \frac{K_{sb}(K_{jc} + K_{sc})}{M_c C_c M_b C_b} \right\} \quad (9)$$

In practical application, the constant current driving is preferable to LED for more stable light output. Therefore, the power input can be approximated by

$$\tilde{P}_d \approx \tilde{V}_f I_f \quad (10)$$

The Eq. (9) is then redefined as

$$\begin{aligned} H_{pj}(s) &\equiv \frac{\tilde{T}_j(s)}{\tilde{I}_f(s)} \\ &= \frac{\bar{V}}{\det(sI - A_{LED})} \left\{ s^3 + \left(\frac{K_{jc} + K_{sc}}{M_c C_c} + \frac{K_{js} + K_{sb} + K_{sc}}{M_s C_s} + \frac{K_{sb}}{M_b C_b} \right) s^2 \right. \\ &\quad + \left[\frac{(K_{jc} + K_{sc})(K_{js} + K_{sb} + K_{sc}) - K_{sc}^2}{M_c C_c M_s C_s} + \frac{K_{sb}(K_{js} + K_{sb} + K_{sc}) - K_{sb}^2}{M_s C_s M_b C_b} \right. \\ &\quad \left. \left. + \frac{K_{sb}(K_{jc} + K_{sc})}{M_c C_c M_b C_b} \right] s + \frac{K_{sb}}{M_c C_c M_s C_s M_b C_b} \left[(K_{jc} + K_{sc})(K_{js} + K_{sc}) - K_{sc}^2 \right] \right\} \quad (11) \end{aligned}$$

The dynamics model $G_{pj}(s)$ needs to be identified.

3. Experimental setup

An LED luminaire was designed and built in the present study (Fig. 4). Five 1 watt PC white LEDs [13] are soldered on a MCPCB to make a lighting module. The module is connected to a copper heat sink on the back side and an aluminum reflector on the front side as the secondary optics. The total weight is 730 g.

The junction temperature of LED cannot be directly measured by any kind of thermometer because LED chip is tiny and packaged. An indirect method, called pulse method, is employed to measure the junction temperature. The methodology is based on fundamental principle of diode PN junction. Eq. (12) is the performance equation for an ideal diode with nondegenerately doping [14].

$$I_f = I_s (e^{qV_f/k_b T_j} - 1) \quad (12)$$

where I_f is forward current, V_f is bias voltage, T_j is junction temperature, I_s is saturation current and q is electric charge. The major thermal sensitivity is due to saturation current I_s . This parameter can be expressed as:

$$I_s = A \cdot q \cdot n_i^2 \cdot \left(\frac{D_h}{N_D L_h} + \frac{D_e}{N_A L_e} \right) \quad (13)$$

where A is the section area of the diode, D_h and D_e are the diffusion coefficients for holes and electrons, L_h and L_e are the diffusion length for the two types of carriers, N_D and N_A are the concentration of donors and acceptors respectively, and n_i is the intrinsic concen-

tration of carriers. The temperature-dependent components are D_h , D_e , L_h , L_e and n_i , but the overall thermal behavior is practically only dependent on the n_i^2 factor. This factor is essentially affected by temperature:

$$n_i^2 = K_i \cdot T_j^3 \cdot e^{-E_{go}/k_b T_j} \quad (14)$$

where K_i is a constant and E_{go} is energy gap of the semiconductor at zero Kelvin. Under typical forward bias conditions, the diode voltage $V_f \gg k_b T_j/e$, and thus $[\exp(qV_f/k_b T_j) - 1] \approx \exp(qV_f/k_b T_j)$. The main effect of T_j also comes from the exponential factor. At a constant biasing current, the voltage decreases with temperature:

$$\left. \frac{dV_f}{dT_j} \right|_{I_f \text{ const.}} \cong - \frac{E_{go} - qV_f}{qT_j} \quad (15)$$

The present study used a single LED lamp to verify this relationship between voltage V_f and junction temperature T_j in a small wind tunnel. The air flow and temperature in the wind tunnel is kept steady. At a steady state, the LED junction temperature is identical with the air flow temperature when no biasing current is applied. In order to avoid the temperature change by diode resistance during the current input, the pulse current with 50 μ s short duration is applied and the resultant voltage is detected. The voltage is measured using oscilloscope. The input current can be varied from 1 mA to 700 mA and the temperature of air flow varied from 40 °C to 90 °C. The experimental results for voltage–current– T_j is shown in Fig. 5.

When the biasing current I_f is low, the voltage varies with temperature linearly. In this condition, the Eq. (15) can be rewritten as:

$$V_j = a_0 + a_1 T_j \quad \text{at low } I_f \quad (16)$$

where a_1 is a constant, called temperature sensitive parameter (TSP) and a_0 is the relative bias voltage. Due to the energy gap shifted by temperature variation, the bias voltage a_0 is not a constant. Eq. (16) cannot be used directly. Based on the linearity of TSP and initial condition, the relation between voltage V_f and temperature T_j can be expressed as:

$$a_1 = \frac{\tilde{V}_j}{T_j} = \frac{V_j - V_{j0}}{T_j - T_{j0}} \Rightarrow T_j = T_{j0} + \frac{V_j - V_{j0}}{a_1} = T_a + \frac{V_j - V_{j0}}{a_1} \quad (17)$$

where V_{j0} is initial voltage and T_{j0} is initial junction temperature which is equal to ambient temperature in thermal equilibrium condition. The junction temperature can be measured by using low bias current (1 mA) at short current pulse (50 μ s).

LEDs is driven with high current for lighting which results in heating and temperature rise. But in measuring the junction tem-

Fig. 4. Schematic of LED luminaire design.

Fig. 5. Variation of forward voltage with junction temperature under different current pulse.

perature, LED should be driven at low current. A high-speed electronic switching device was thus designed in the present study for measuring the instantaneous junction temperature response during dynamic testing of the LED luminaire. The short current dropped duration (50 μ s) can minimize the distortion of thermal response [15].

4. System identification

The system dynamics model, Eq. (11), can be identified using step response test method. A step current input was applied to the LED luminaire and the responses of junction temperature was recorded. Repeating the experiment at different current steps, we can determine the linearly-perturbed model at various operating conditions. The transient junction temperature measurement using the aforementioned method was applied to this experiment. The range of input current to each LED lamp is between 100 and 350 mA. The system dynamic tests were performed with step change 50 mA. The luminaire was placed at ambient with natural convection.

The time response data was used to analyse the system dynamics model using Rake's method [16]. Rake's method is a time domain curve fitting technique for a step response test. The dynamic model can be obtained by fitting the time response data to a presumed model. Based on Eq. (11), 4th-order system is adopted for identification as Eq. (18). The 4th-order model can fit the experimental data very well. The results at various operating conditions was shown in Table 1. The fitting model and the real response of junction temperature under input current 200 mA to 250 mA are shown in Fig. 6. For simplification in application, all parameters of each operating point can be averaged to represent the average model (Fig. 7). The practical model is identified in Eq. (19).

$$H_4(s) = \frac{k(s+z_1)(s+z_2)(s+z_3)}{(s+p_1)(s+p_2)(s+p_3)(s+p_4)} \quad (18)$$

$$\bar{H}_4(s) = \frac{0.0557(s+0.0031)(s^2+0.0032s+4.141 \times 10^{-6})}{(s+0.2986)(s+0.0023)(s^2+0.0035s+4.601 \times 10^{-6})} \quad (19)$$

The system dynamics of the LED luminaire can be described by four poles and three zeros. The poles are composed of two real and one pair of conjugate roots, and the zeros are formed in one real and one pair of conjugate roots. The distance from imaginary axis to the nearest left pole is 130 times far than others. Obviously, the fast mode (the nearest left pole) can be neglected according to the classic control theorem. Additionally, both of the conjugate pole and zero pairs can be canceled each other since the pole-zero positions are very close. The system can be therefore simplified to a first-order bi-proper (one pole with one zero) system. This simple model can simplify the control system design (see Fig. 8).

Table 1

Poles, zeros and gains of 4th-order model under different driving current perturbation

No.	Step I_f (mA)	p_1	p_2	Real part of p_3, p_4	Imaginary part of p_3, p_4	z_1	Real part of z_2, z_3	Imaginary part of z_2, z_3	k
1	100–150	−0.4084	−0.0021	−0.0015	0.0014	−0.0028	−0.0015	0.0013	0.0796
2	150–200	−0.2976	−0.0018	−0.0012	0.0007	−0.0022	−0.0013	0.0007	0.0565
3	200–250	−0.2464	−0.0029	−0.0017	0.0009	−0.0027	−0.0019	0.0010	0.0403
4	250–300	−0.3775	−0.0019	−0.0009	0.0013	−0.0023	−0.0009	0.0013	0.0696
5	300–350	−0.2453	−0.0025	−0.0022	0.0014	−0.0037	−0.0019	0.0015	0.0433
6	350–300	−0.3037	−0.0026	−0.0022	0.0015	−0.0037	−0.0019	0.0015	0.0534
7	300–250	−0.2698	−0.0025	−0.0021	0.0014	−0.0037	−0.0018	0.0015	0.0498
8	250–200	−0.2859	−0.0022	−0.0020	0.0011	−0.0038	−0.0015	0.0011	0.0529
9	200–150	−0.2718	−0.0021	−0.0018	0.0012	−0.0032	−0.0015	0.0012	0.0539
10	150–100	−0.2792	−0.0024	−0.0019	0.0014	−0.0033	−0.0017	0.0014	0.0579
Average model		−0.2986	−0.0023	−0.0018	0.0012	−0.0031	−0.0016	0.0013	0.0557

Fig. 6. Comparison of step response of junction temperature using input current from 200 to 250 mA.

Fig. 7. Step response of junction temperature for various 4th-order perturbed model and the average model using 50 mA input current step.

We identify system dynamics again using the first-order bi-proper system model. The result also can fit the experimental data very well. For the step input from 200 to 250 mA, the results are

Fig. 8. The pole-zero positions of 4th-order system in s-plane.

Fig. 9. Junction temperature change with input current step from 200 to 250 mA.

Table 2
Poles, zeros and gains of first-order model under different driving current perturbation

No.	Step $\bar{I}_f$ (mA)	p	z	k
1	100–150	−0.001874	−0.002254	0.198075
2	150–200	−0.001789	−0.002253	0.186865
3	200–250	−0.001612	−0.002029	0.183383
4	250–300	−0.001624	−0.001954	0.186878
5	300–350	−0.001790	−0.002246	0.176255
6	350–300	−0.001553	−0.001939	0.176242
7	300–250	−0.001579	−0.001939	0.185754
8	250–200	−0.001551	−0.001882	0.192187
9	200–150	−0.001567	−0.001887	0.197576
10	150–100	−0.001526	−0.001837	0.211636
Average model		−0.001647	−0.002022	0.189485

shown in Fig. 9. All the perturbed and average model are listed in Table 2 and shown in Fig. 10. Thus, the average model is shown in Eq. (20).

$$\bar{H}_1(s) = \frac{0.18949(s + 0.002022)}{s + 0.001647} \quad (20)$$

Fig. 10. Junction temperature response for various first-order model and its average model using 50 mA input current step.

5. Result and discussion

5.1. Dynamic heat transfer mechanism of LED luminaire

According to the system dynamics of 4th-order model in Eq. (18), the two real poles represent two heat transfer modes. Both modes can describe as two time constants (τ_1 and τ_2). The thermal time constant is an important characteristic which determines the transient thermal behavior of specific element [17]. The time constants of the average model in Eq. (19) is

$$\tau_1 = \frac{1}{p_1} = \frac{1}{0.2986} = 3.35 \text{ s} \quad (\text{fast mode}) \quad (21)$$

$$\tau_2 = \frac{1}{p_2} = \frac{1}{0.0023} = 435.1 \text{ s} \quad (\text{slow mode}) \quad (22)$$

The fast mode is much faster than the slow mode. This phenomenon can be explained from the heat transfer mechanism of LED. The chip temperature rise is dominated by the heat transfer from the chip to its adjacent element (capsule and heat slug) at the beginning. The small contact surface or low conductivity of chip becomes a thermal barrier. The chip temperature is therefore raised rapidly [18]. After that, the heat transfer to heat sink will dominate the response. The chip temperature will rise slowly due to large thermal capacity of the luminaire. Therefore, the fast mode can be treated as a result of junction to heat slug heat transfer. The slow mode is related to the heat transfer to the heat sink. Similarly, the pair of conjugate mode (p_3 and p_4) is caused by the heat interaction between capsule and heat slug. The low heat conductance of capsule and small mass of heat slug make the dynamics of the pair of conjugate mode, but this phenomenon may be too small and can be ignored. This results in a pole-zero cancellation. Consequently, the system dynamics can be reduced to first-order biproper system.

Moreover, the first-order bi-proper model can be divided into a constant two portions: a constant γ and a first-order model $H_{th}(s)$. The average model can describe in this form in Eq. (23).

$$H_1(s) = \frac{0.18949(s + 0.002022)}{s + 0.001647} \equiv \bar{H}_{th}(s) + \bar{\gamma} = \frac{7.10 \times 10^{-5}}{s + 0.001647} + 0.1895, \gamma \in R$$

where $\bar{H}_{th}(s) = \frac{7.10 \times 10^{-5}}{s + 0.001647}$, $\bar{\gamma} = 0.1895 \text{ unit} : \frac{^\circ\text{C}}{\text{mA}}$ (23)

Refer to Fig. 9, the dynamic response leads to an instantaneous jump at initial state. The jump is due to the pole-zero cancellation of Eq. (19) and will lose model accuracy at initial 50 s. This error is negligible compared to the settling time about 2200 s. After that, the model of heat sink $H_{th}(s)$ fits the response data well.

The steady-state gain of $H_{th}(s)$ in Eq. (24), $0.0431\text{ }^{\circ}\text{C}/\text{mA}$, reveals that it is about quarter of instantaneous jump value ($0.1895\text{ }^{\circ}\text{C}/\text{mA}$). This indicates that the initial jump dominates the dynamic response of junction temperature due to poor heat transfer from chip.

The system dynamics model indicates that the step response has no overshoot and the steady-state gain of $H_1(s)$ is $0.2326\text{ }^{\circ}\text{C}/\text{mA}$ given by Eq. (25). This means that, at ambient temperature $25\text{ }^{\circ}\text{C}$, the junction temperature will not exceed the limit ($120\text{ }^{\circ}\text{C}$) if the input current step is smaller than 400 mA for each LED lamp.

$$|H_{th}(s)| = H_{th}(s=0) = \lim_{s \rightarrow 0} \frac{7.10 \times 10^{-5}}{s + 0.001647} = 0.0431\text{ }^{\circ}\text{C}/\text{mA} \quad (24)$$

$$|H_u(s)| = H_u(s=0) = \lim_{s \rightarrow 0} \frac{0.18949(s + 0.002022)}{s + 0.001647} = 0.2326\text{ }^{\circ}\text{C}/\text{mA} \quad (25)$$

$$T_j(s) = (H_{th}(s) + \gamma) \cdot I_f(s) \quad (26)$$

Based on the heat transfer mechanism of an LED luminaire, the information-flow diagram for the heat transfer in an LED luminaire can be drawn as Fig. 11. The junction temperature variation of an LED luminaire is resulted from the input current and the ambient temperature. The block diagram of the system dynamics is shown in Fig. 12. The junction temperature response under current input can also be expressed as Eq. (26).

5.2. Electric-heat-optical effect of LED luminaire

The above discussion is only for the heat transfer mechanism (energy effect) in an LED luminaire. In practice, the luminance of LED is generated from two physical mechanisms: the energy effect according to the heat transfer principle and the optoelectronic effect according to the principle of solid-state lighting. The energy effect can be described using the system dynamic model, Fig. 12. The

Fig. 11. The information-flow diagram of LED luminaire.

Fig. 12. Practical first-order model of LED luminaire.

Fig. 13. Electric-heat-optical effect of LED luminaire.

optoelectronic effect is generated mainly from the current input to LED. However, the junction temperature will also affect the light output of a LED lamp. Assuming the influence of junction temperature and power input on light output (luminance) Φ is in linear relationship [19], the responses of luminance can be expressed as Eq. (23), in which δ and ε are proportional constants.

$$\begin{aligned} \Phi(s) &= \delta \cdot T_j(s) + \varepsilon \cdot I_f(s) = [\delta \cdot (H_{th}(s) + \gamma) + \varepsilon] \cdot I_f(s) \\ &= (\delta \cdot H_{th}(s) + \mu) \cdot I_f(s) \end{aligned} \quad (27)$$

Eq. (27) is the general physical model of an LED luminaire which describes the complete physical phenomenon of an LED lighting process, an electric-heat-optical effect, which can be schematically shown in Fig. 13. This general LED model can be used for the study of LED lighting control.

6. Conclusions

Optical properties of LED are sensitive to junction temperature as well as input current. The understanding of system dynamics behavior is quite important in lighting control design of LED. From the principle of solid-state lighting, the luminance of LED is generated from two physical mechanisms: energy effect and optoelectronic effect.

The system dynamics model for energy effect is derived analytically using the principle of energy conservation to each part of LED luminaire. The model is then identified using step response test method. Both theoretical and experimental analyses have shown that the system dynamics model of LED fixture is 4th-order with three zeros and can be further reduced to a first-order biproper system (one pole with one zero). It is shown that the instantaneous jump of junction temperature dominates the thermal behavior of LED at the beginning of step input.

According to the system dynamics of 4th-order model in Eq. (14), the two time constants represent two modes, fast mode and slow mode. The fast mode can be treated as a result of junction and heat slug heat transfer. The slow mode is related to the heat sink heat transfer. For the LED lighting fixture used in the present study, the system dynamics model indicates that at ambient temperature $25\text{ }^{\circ}\text{C}$, the junction temperature will not exceed the limit ($120\text{ }^{\circ}\text{C}$) if the input current is smaller than 400 mA for each LED lamp.

The optoelectronic effect of a LED is induced mainly from the current input and the junction temperature. An electric-heat-optical system dynamics model of LED luminaire was finally derived which described the complete physical phenomenon of an LED lighting process, an electric-heat-optical effect.

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